EGR 1101 Laboratory Notebook

Name: ____________________________________

Year: __________ Semester______________

TA: ____________________________Section:__________

This page left in on purpose.

Grade Sheet

Application of Algebra in Engineering: ______ /30

Application of Trigonometry in Engineering: ______ /28

Properties and Manipulations of Sinusoids: ______ /28

Two Loop Circuit Application of Systems of Equations: ______ /16

Freefall Application of the Derivative: ______ /22

Spring Work Application of the Integral: ______ /38

Leaking Bucket:
Application of a First Order Differential Equation: ______ /20

Spring-Mass:
Application of a Second Order Differential Equation: ______ /16

Laboratory 1

Application of Algebra in Engineering

1.1 Laboratory Objective

The objective of this laboratory is to illustrate linear and quadratic applications utilized in engineering. Supplementary information includes basic MATLAB commands and functions.

1.2 Educational Objectives

After performing this experiment, students should be able to:

1. Perform basic algebraic manipulations with linear equations.
2. Perform basic algebraic manipulations with quadratic equations.
3. Measure and understand the relationship between voltage, current, and resistance.
4. Apply basic functions in MATLAB toward the solution of engineering equations using the command window.
5. Use MATLAB for plotting data.

1.3 Background

It is essential all engineers have an understanding of the fundamental laws of electricity. Ohm's Law and Kirchhoff's Voltage Law are two such laws that are presented in this lab. In addition, knowledge of the equipment and instrumentation that employ these laws is comparably important. Some of these include ammeters, voltmeters, watt-meters, breadboards, and circuitry components such as resistors. The implementation of these instruments are introduced in this lab.

1.3.1 Ohm's Law

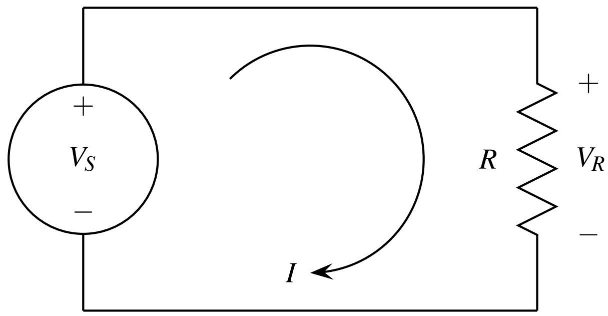

Figure 1.1: An Electrical Circuit Consisting of a Voltage Source V_S and Resistive Element R

Ohm's Law is a linear equation stating the voltage across a resistor is equal to the current flowing through that resistor multiplied by the value of that resistor. The following equation relates to Figure 1.1:

$$V_R = IR \tag{1.1}$$

The value V_R is the voltage across the resistor in volts (V), I is the current flowing through the resistor in amperes (A), and R is the resistance in ohms (Ω).

1.3.2 Kirchhoff's Voltage Law

Kirchhoff's Voltage Law states that the sum of the voltage rises is equal to the sum of the voltage drops in a circuit.

$$\Sigma\text{Voltage Rises} = \Sigma\text{Voltage Drops}$$

Therefore, for the circuit shown in Figure 1.1:

$$V_S = V_R = IR \tag{1.2}$$

1.3.3 Equipment

A breadboard and resistors are two electrical components presented in this lab. The function of breadboards along with identification of resistor values will be reviewed. In addition, three types of measuring devices are introduced in this lab. These include an ammeter, voltmeter, and watt-meter. Lastly, multiple power supplies will be utilized in the circuit construction.

1.3.3.1 Breadboard

A breadboard is a medium to prototype a circuit. Circuit components are attached to the breadboard by inserting wires or leads into the small holes arranged in grids on the board. Since these components are not soldered in place, the pieces can be removed and the circuit easily changed. A standard breadboard is shown in Figure 1.2.

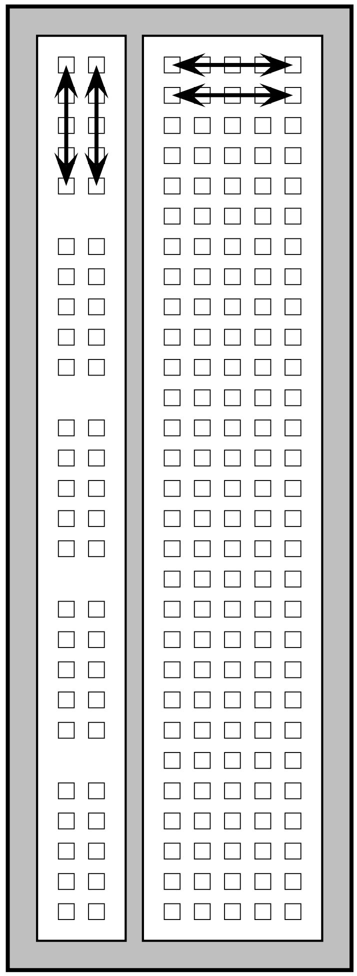

Figure 1.2: A Standard Breadboard Layout

Inside the breadboard are metal contacts that connect the holes. These metal contacts join clusters of five holes together and are connected per the arrows shown in Figure 1.2. These clusters of five holes can be considered as one node.

1.3.3.2 Resistors

Resistors are electrical components that dissipate power by consuming current. This enables engineers to regulate the amount of current allowed to flow into succeeding components in the circuit. All resistors have a maximum power limit. The tiny resistors used in lab are quarter watt resistors and the larger ones are ten watt resistors. Because of physical size limitations for printing, a standard for defining resistor values has been developed. For the larger resistors, the value is printed right on the casing. This standard uses color coded bands that in conjunction with a chart, yield the resistor value. Figure 1.3 shows an example of a typical resistor defined by colored bands.

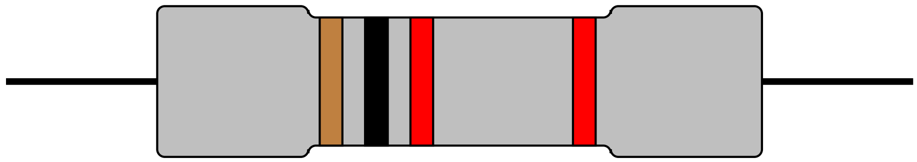

Figure 1.3: A 1000Ω Resistor

The colored bands of this resistor correspond to Table 1.1 and Table 1.2. Reading from left to right, the first two bands give the first two digits of the resistor value. The third colored band is the multiplier. This value tells to what power of ten we multiply the first two digits. The resistor value in Figure 1.3 is found using Tables 1.1 and 1.2 as follows:

- The Brown band corresponds to a 1.
- The Black band corresponds to a 0.
- The Red band indicates multiplying by 10^2.
- The next Red band indicates a tolerance of ±2%.

The value of this resistor is 10 times 10^2 resulting in 1000Ω with a tolerance of ±2%.

Table 1.1 Resistor Color Band Values

Number	0	1	2	3	4	5	6	7	8	9
Color	Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Grey	White

Table 1.2 Resistor Tolerance Color Band Values

Tolerance	±1%	±2%	±5%	±10%
Color	Brown	Red	Gold	Silver

1.3.3.3 Ammeter

An ammeter is a device that measures the current flowing in a circuit. Because it measures a quantity moving through the circuit it must be connected in series as shown in Figure 1.4.

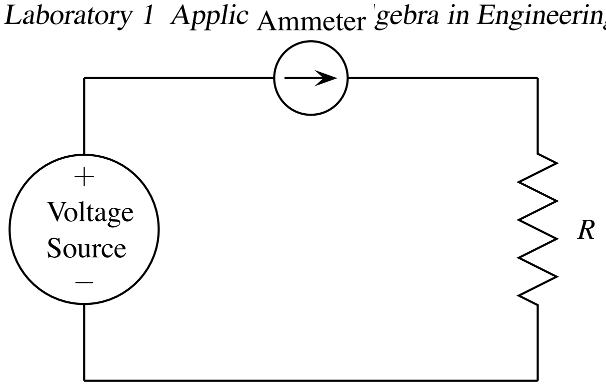

Figure 1.4: Placement of an Ammeter in a Circuit

1.3.3.4 Voltmeter

A voltmeter is a device that measures the voltage potential across an electrical component. Because of this, it is placed in parallel with the component whose voltage drop is being measured. A voltmeter is used to measure the voltage drop across the resistor in Figure 1.5.

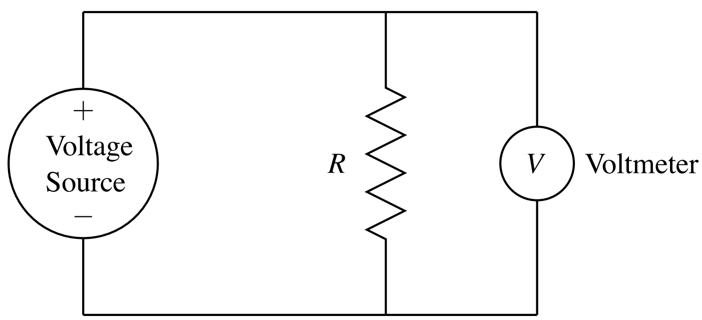

Figure 1.5: Placement of a Voltmeter in a Circuit

1.3.3.5 Watt-meter

A watt-meter is a device that measures the power used by an electrical component. The power delivered or absorbed is given by some basic equations related by Ohm's Law:

$$P = VI = I^2R = \frac{V^2}{R} \tag{1.3}$$

A watt-meter functions by simultaneously measuring the current passing through, and the voltage drop across, the component. In practice, this requires four connections to a circuit. The current nodes will be connected in series while the voltage nodes will be connected in parallel. The watt-meter that is connected in Figure 1.6 is set up to measure the power dissipated by the resistor R.

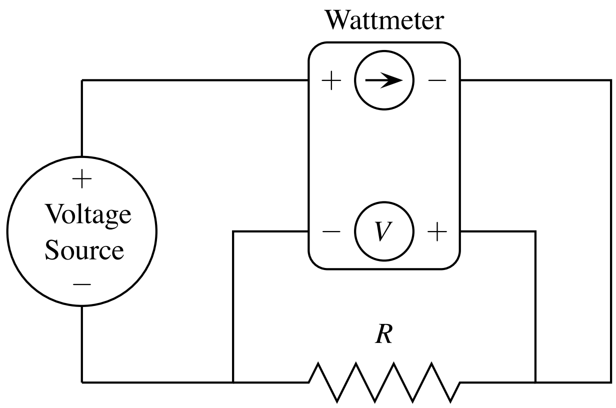

Figure 1.6: Placement of a Wattmeter in a Circuit

1.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the laboratory equipment.

1.4.1 Circuit Number 1

1. The value of the resistor in Figure 1.7 is unknown. Construct the circuit with a quarter watt resistor and use the laboratory equipment to find this value. Complete Table 1.3 and record the current value measured on the ammeter.

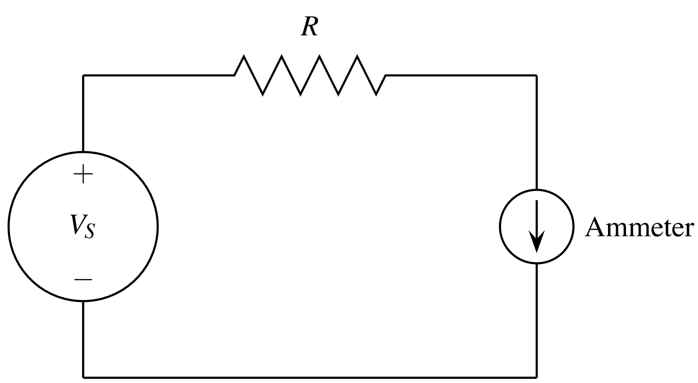

Figure 1.7: Circuit for Section 1.4.1

Table 1.3 Circuit 1 Measurements

Voltage V_S (volts)	Measured Current I (amps)	Calculated Resistance R (Ω)
0	0	0
4		
8		
12		
15		

2. Calculate the resistance R in the last column using Ohm's Law. (Pay close attention to units!)

$$R = \frac{V_S}{I}$$

3. Attach these hand calculations at the end of the lab.
4. Plot V_S vs. I using MATLAB.

   NOTE: Standard notation is y vs. x. Place V_S on the y-axis and I on the x-axis.

   NOTE: Use the MATLAB syntax that is given in Section 1.5.
5. Using MATLAB's Basic Fitting tool, find the slope of the graph.
6. Print the graph.

1.4.2 Circuit Number 2

1. The value of the quarter watt resistor in Figure 1.8 below is unknown. Construct the circuit and use the laboratory equipment to find this value. Complete Table 1.4 and record the current value measured on the ammeter.

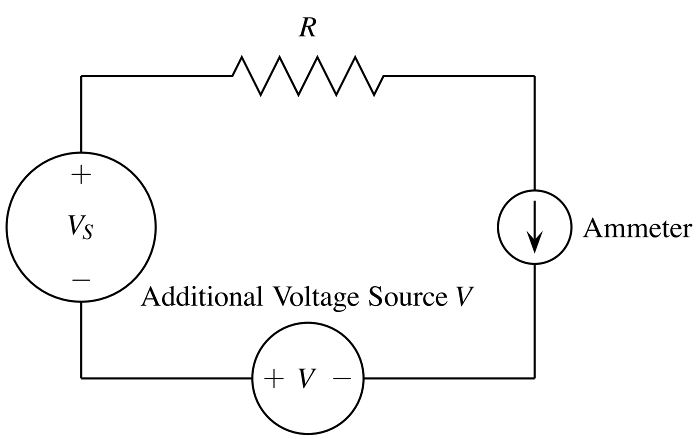

Figure 1.8: Circuit for Section 1.4.2

Table 1.4 Circuit 2 Measurements

Voltage V_S (volts)	Add. Voltage V (volts)	Measured Current I (amps)	Calculated Resistance R (Ω)
0	5		
5	5		
7	5		
9	5		

2. Calculate the resistance R in the last column using Ohm's Law. (Pay close attention to UNITS!)

$$R = \frac{V_S + V}{I}$$

3. Attach these hand calculations at the end of the lab.
4. Plot V_S vs. I in MATLAB using the syntax in Section 1.5.
5. Using MATLAB's Basic Fitting tool, find the slope and y-intercept of the graph.
6. Print the graph.

1.4.3 Circuit Number 3

1. The value of the current flowing through the circuit in Figure 1.9 is unknown. Construct the circuit using *ten watt resistors* and use the laboratory equipment to find it. Complete Table 1.5 and record the power and current from the watt-meter.

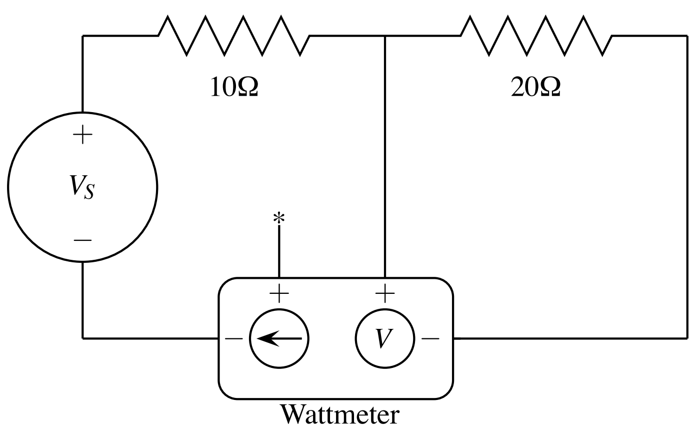

Figure 1.9: Circuit for Section 1.4.3

Table 1.5 Circuit Three Measurements

Voltage V_S (volts)	Power P (watts)	Measured Current I (amps)	Calculated Current I_{calc} (amps)
7			
9			
11			

2. Use the following quadratic equation to calculate the theoretical current and record that value in the last column in Table 1.5. (Pay close attention to UNITS!)

$$RI_{calc}^2 - V_S I_{calc} + P = 0$$

NOTE: The value for R in this equation is 10, *not* 20. P in this equation is the power dissipated by the 20Ω resistor.

3. Attach these hand calculations at the end of the lab.

1.5 MATLAB Commands

x = [ ]; This command defines a row vector x. Place real numbers within the square brackets separated by spaces or commas.

plot (x , y , ’o’) This command plots the data in vectors x and y and does not connect lines between each point.

To fit a curve to the data:

1. On the figure, go to the “Tools” drop down menu.
2. Highlight “Basic Fitting”.
3. Check the “Linear” box.
4. Check the “Show Equations” box.

This will fit a linear curve to the data and place the equation of that line on the plot.

1.6 Lab Requirements

1. Complete Tables 1.3, 1.4, and 1.5. (2 points each)
2. Write an abstract for this lab. Insert after this page. (Writing grade: 2 points)
3. Show hand calculations for all three tables. Insert after this page. (2 points each)
4. Insert both plots after this page. (Don't forget axis labels and title!) (2 points each)
5. Answer the following questions.

   a) To what component of circuit one does the slope of plot one correspond? (2 points)

   b) To what component of circuit two does the y-intercept of plot two correspond? (2 points)

   c) Refer to circuits one and two for the following questions:

      i. For circuits one and two, the calculated R should be relatively close to what value? (2 points)

      ii. By how much do these values differ from the theoretical resistance as a percentage (calculate for the maximum voltage case of circuits #1 and #2)? Show work. (2 points)

      iii. Is this within the tolerance of the resistor? (2 points)

   d) How do the values for the measured current and calculated current from circuit three compare? What are some reasons for this? (2 points)

Laboratory 2

Application of Trigonometry in Engineering

2.1 Laboratory Objective

The objective of this laboratory is to learn basic trigonometric functions, conversion from rectangular to polar form, and vice-versa.

2.2 Educational Objectives

After performing this experiment, students should be able to:

1. Understand the basic trigonometric functions.
2. Understand the concept of a unit circle and four quadrants.
3. Understand the concept of a reference angle.
4. Be able to perform the polar to rectangular and rectangular to polar coordinate conversion.
5. Prove a few of the basic trigonometric identities.

2.3 Background

Trigonometry is a tool that mathematically forms geometrical relationships. The understanding and application of these relationships are vital for all engineering disciplines. Relevant applications include automotive, aerospace, robotics, and building design. This lab will outline a few common, but useful, trigonometric relationships.

2.3.1 Reference Angle

A reference angle is an acute angle (less than 90°) that may be used to compute the trigonometric functions of the corresponding obtuse angle (greater than 90°). Figure 2.1 shows the reference angle ϕ with respect to the angle θ.

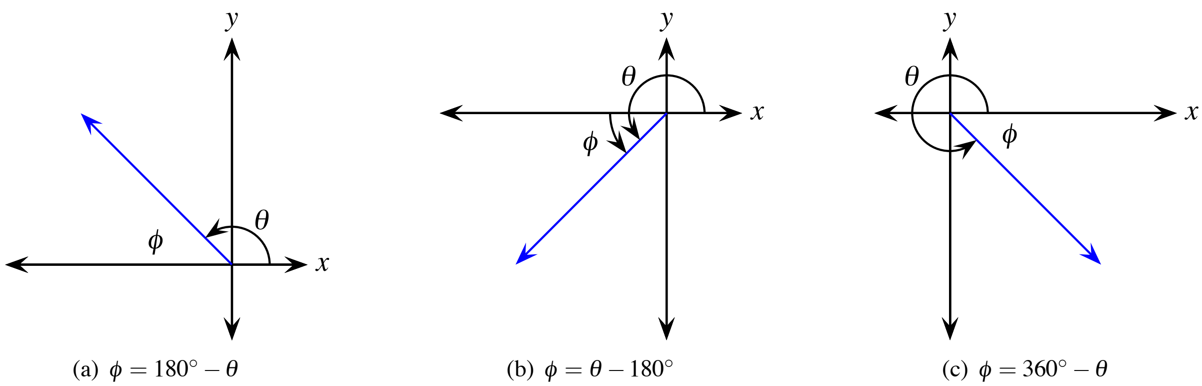

Figure 2.1: Reference Angle Calculations in Different Quadrants

The reference angle ϕ is calculated using the formulas shown in the captions of each corresponding subfigure of Figure 2.1.

2.3.2 Law of Cosines

The law of cosines is a method that helps to solve triangles. Equations 2.1 relates the sides and interior angles of Figure 2.2.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos(A) \\ b^2 &= a^2 + c^2 - 2ac\cos(B) \\ c^2 &= a^2 + b^2 - 2ab\cos(C) \end{aligned} \tag{2.1}$$

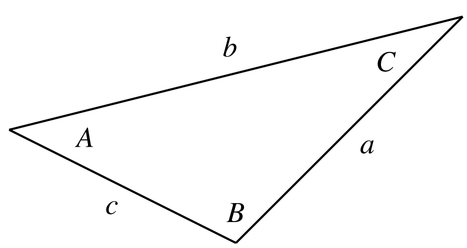

Figure 2.2: Law of Cosines Triangle

2.3.3 Law of Sines

The Law of Sines is another method that helps to solve triangles. Using the triangle of Figure 2.2, Equation 2.2 relates the sides to the interior angles.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \tag{2.2}$$

2.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the laboratory equipment.

2.4.1 One Link Robot

1. Using the boards in the lab, fill in Table 2.1. Pay close attention to the sign of your answer for all values.

   NOTE: To convert a value in degrees to radians, the multiplying factor is $\frac{\pi}{180}$.

2. Use Equations 2.3 to find the Calculated x and y values.

$$x = l\cos(\theta) \tag{2.3}$$

$$y = l\sin(\theta)$$

Table 2.1 Polar to Rectangular Conversion

Angle θ (°)	Measured x (mm)	Measured y (mm)	Vector Form $x\hat{i}+y\hat{j}$	l (mm)	Reference Angle (°)	Reference Angle (radians)	Calculated x (mm)	Calculated y (mm)
30				100				
45				100				
90				100				
135				100				
180				100				
225				100				
270				100				

3. Using the boards in the lab, fill in Table 2.2.

4. Use Equations 2.4 to find the Calculated θ and l.

$$\theta = \tan^{-1}(y/x) \tag{2.4}$$

$$l = \sqrt{x^2 + y^2}$$

Table 2.2 Rectangular to Polar Conversion

(x,y)	Measured θ (°)	Reference Angle (°)	Reference Angle (radians)	Calculated θ (°)	Calculated l (mm)	Polar Form $l\measuredangle\theta$
$(85,50)$						
$(70,70)$						
$(0,100)$						
$(-70,70)$						
$(-100,0)$						
$(-70,-70)$						

2.4.2 Identity Verification

An identity is a trigonometric relationship that is true for all permissible values of the variable(s). Many times, trigonometric identities are used to simplify more complex problems.

1. Using MATLAB, fill in Tables 2.3 and 2.4.

    a) The first column of Table 2.3 comes from Table 2.2.

    b) Define this column as a vector in MATLAB and perform element by element calculations on it to get the other columns.

NOTE: All calculations should be done with MATLAB. No calculator use!

Table 2.3

Calculated θ from Table 2.2 (°)	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\sec(\theta)$

Table 2.4

$\frac{\sin(\theta)}{\cos(\theta)}$	$\sin^2(\theta)+\cos^2(\theta)$	$1+\tan^2(\theta)$	$\sec^2(\theta)$

2.4.3 Two Link Robot

1. Using the boards in the lab, fill in the Measured Values of Table 2.5.
2. Write a MATLAB code to calculate x and y by adding the components of each link. Recall the following equations from class.

$$x_1 = l_1 \cos(\theta_1)$$

$$y_1 = l_1 \sin(\theta_1)$$

$$x_2 = l_2 \cos(\theta_1 + \theta_2)$$

$$y_2 = l_2 \sin(\theta_1 + \theta_2)$$

$$X = x_1 + x_2$$

$$Y = y_1 + y_2$$

Table 2.5

		Measured Values						Calculated Values	
$\theta_1(^\circ)$	$\theta_2(^\circ)$	x_1	y_1	x_2	y_2	$X=x_1+x_2$	$Y=y_1+y_2$	X	Y
0	0								
0	90								
30	45								
30	60								
180	0								
270	30								
360	90								

2.4.4 Solve a Triangle Using Law of Cosines and Law of Sines

In some cases, the laws of sines and cosines must both be used to solve a triangle. Figure 2.3 is one such case where the lengths l_1 and l_2 along with the final ending point P of the two links are known and the θ values are not. Both laws are needed to solve this triangle.

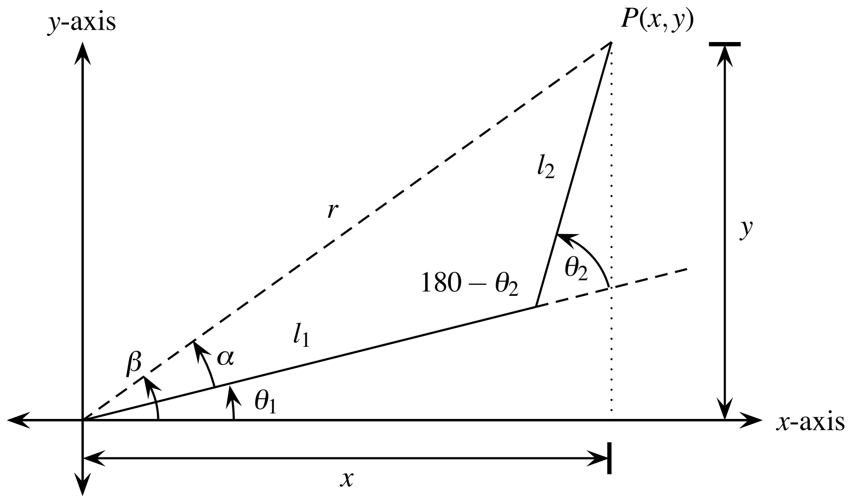

Figure 2.3: General Two Link Robot

1. The radius r is found by:

$$r = \sqrt{x^2 + y^2} \tag{2.5}$$

2. Using the Law of Cosines, θ_2 is found by the following equation:

$$r^2 = l_1^2 + l_2^2 - 2 l_1 l_2 \cos(180 - \theta_2)$$

$$\theta_2 = 180 - \cos^{-1}\left(\frac{r^2 - l_1^2 - l_2^2}{-2 l_1 l_2}\right) \tag{2.6}$$

3. Using the Law of Sines, α is found by the following equation:

$$\frac{r}{\sin(\theta_2)} = \frac{l_2}{\sin(\alpha)}$$

$$\alpha = \sin^{-1}\left(\frac{l_2 \sin(\theta_2)}{r}\right) \tag{2.7}$$

4. θ_1 is now found by the equation:

$$\beta = \tan^{-1}\left(\frac{y}{x}\right) \tag{2.8}$$

$$\theta_1 = \beta - \alpha \tag{2.9}$$

5. Using Equations 2.5, 2.6, 2.7, 2.8, and 2.9, write a MATLAB code to fill in Table 2.6.

   NOTE: $l_1 = l_2 = 50$mm.

NOTE: Define x and y as vectors containing all points below.

Table 2.6 Application of Sine and Cosine Laws

$P(x,y)$	$\theta_2(^\circ)$	$\alpha(^\circ)$	$\beta(^\circ)$	$\theta_1(^\circ)$
(55,75)				
(75,60)				
(15,63)				
(32,14)				
(71,70)				

2.5 Lab Requirements

1. Complete Tables 2.1, 2.2, 2.3, 2.4, 2.5, and 2.6. (2 points each)
2. Write an abstract for this lab. Insert after this page. (Writing grade: 2 points)
3. Show hand calculations for Tables 2.1 and 2.2. Insert after this page. (2 points each)
4. MATLAB for Tables 2.3 and 2.4. (2 points each)
   a) m-file
   b) output
5. MATLAB for Table 2.5. (2 points)
   a) m-file
   b) output
6. MATLAB for Table 2.6. (2 points)
   a) m-file
   b) output
7. Answer the following questions.
   a) Based on your results for Tables 2.3 and 2.4, write down the three trigonometric identities that were verified. (2 points)

Laboratory 3

Sinusoids in Engineering: Measurement and Analysis of Harmonic Signals

3.1 Laboratory Objective

The objective of this laboratory is to understand the basic properties of sinusoids and sinusoid measurements.

3.2 Educational Objectives

After performing this experiment, students should be able to:

1. Understand the properties of sinusoids.
2. Understand sinusoidal addition.
3. Obtain measurements using an oscilloscope.

3.3 Background

Sinusoids are sine or cosine waveforms that can describe many engineering phenomena. Any oscillatory motion can be described using sinusoids. Many types of electrical signals such as square, triangle, and saw-tooth waves are modeled using sinusoids. Their manipulation incurs the understanding of certain quantities that describe sinusoidal behavior. These quantities are described below.

3.3.1 Sinusoid Characteristics

Amplitude The amplitude A of a sine wave describes the height of the hills and valleys of a sinusoid. It carries the physical units of what the sinusoid is describing (volts, amps, meters, etc.).

Frequency There are two types of frequencies that can describe a sinusoid. The normal frequency f is how many times the sinusoid repeats per unit time. It has units of cycles per second or Hertz (Hz). The angular frequency ω is how many radians pass per second. Consequently, ω has units of radians per second.

Period The period T is how long a sinusoid takes to repeat one complete cycle. The period is measured in seconds.

Phase The phase φ of a sinusoid causes a horizontal shift along the t-axis. The phase has units of radians.

TimeShift The time shift t_s of a sinusoid is a horizontal shift along the t-axis and is a time measurement of the phase. The time shift has units of seconds.

NOTE: A sine wave and a cosine wave only differ by a phase shift of 90º or $\frac{\pi}{2}$ radians. In reality, they are the same waveform but with a different φ value.

3.3.2 Sinusoidal Relationships

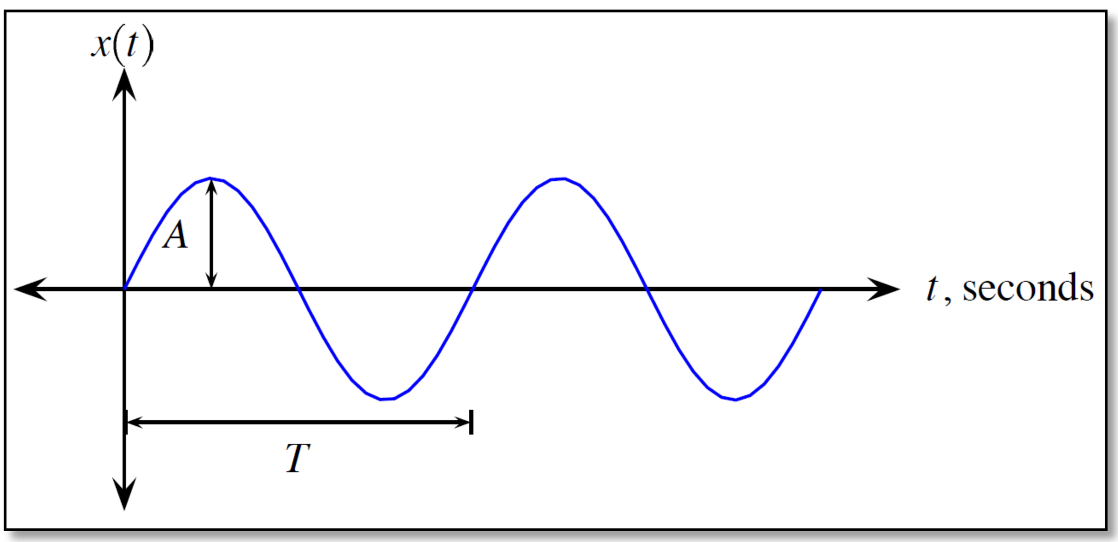

Figure 3.1: Sinusoid.

The general equation of a sinusoid is given below and refers to Figure 3.1.

$$x(t) = Asin(\omega t + \varphi) \tag{3.1}$$

The angular frequency is related to the normal frequency by Equation 3.2.

The angular frequency is related to the normal frequency by Equation 3.2.

$$\omega = 2\pi f \tag{3.2}$$

The angular frequency is also related to the period by Equation 3.3.

$$\omega = \frac{2\pi}{T} \tag{3.3}$$

By inspection, the normal frequency is related to the period by equation 3.4

$$f = \frac{1}{T} \tag{3.4}$$

The time shift is related to the phase (radians) and the frequency by Equation 3.5.

$$t_s = \frac{\varphi}{2\pi f} \tag{3.5}$$

3.3.3 Sinusoidal Measurements

1. Connect the output channel of the Function Generator to channel one of the Oscilloscope.
2. Complete Table 3.1 using the given values for voltage and frequency. Attach hand calculations at the end of the lab.

Table 3.1 Sinusoid Measurements

Function Generator		Oscilloscope (Measured)			Calculated	
Amplitude (V)	***Frequency (Hz)***	$A\left(\frac{V_{P-P}}{2}\right)$	***f (Hz)***	***T(sec)***	***ω(rad/sec)***	***T(sec)***
½ MAX	**1000**					
MAX	**5000**					

3. Using an Oscilloscope, make measurements across the two separate resistors and complete **Table 3.2** use ***f=2500 Hz***.
   a. Connect Channel 1 of the oscilloscope as shown in **Fig. 3.2A** and measure the amplitude of the resistor signal that is in series with the capacitor. (NOTE: Polarity is important; all the negative connections should be hooked together when making measurements.)
   b. Move Channel 1 of the oscilloscope as shown in **Fig. 3.2B** and measure the amplitude of the resistor signal that is in series with the inductor.

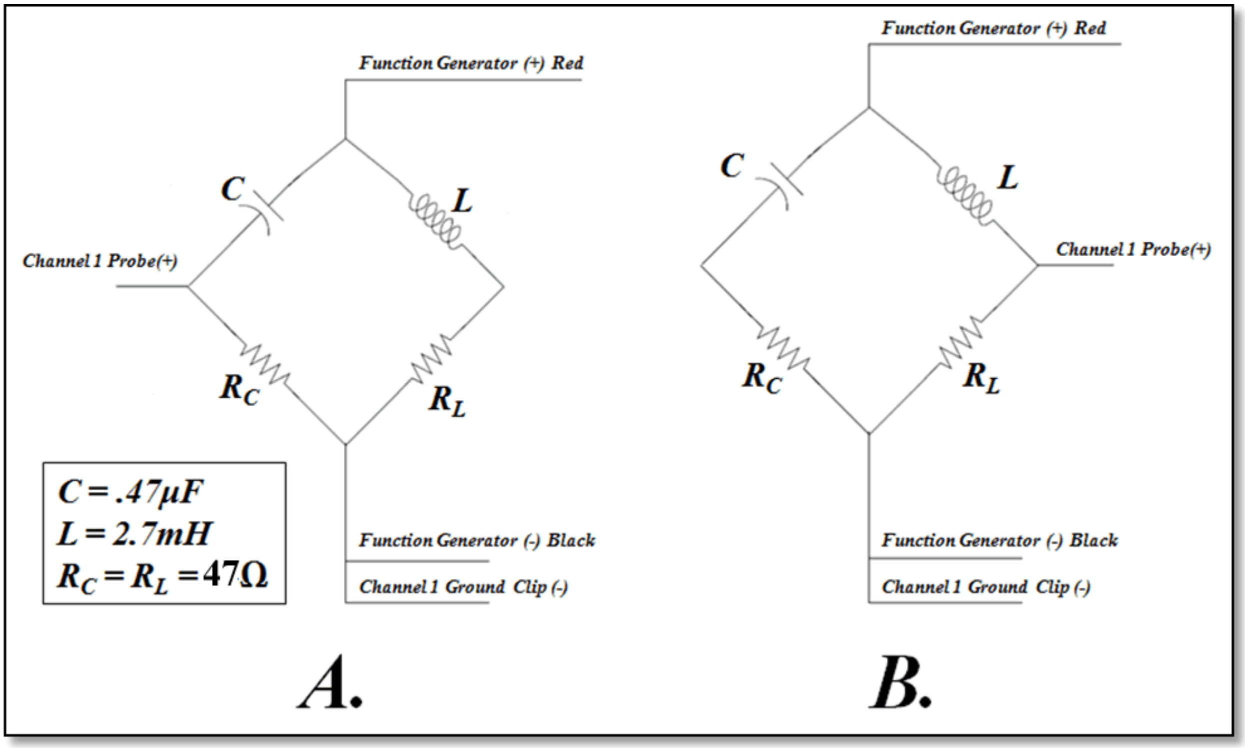

Figure 3.2: Voltage Measurement.

Table 3.2 Sinusoid Amplitude Measurements

Signal	Amplitude (*Volts*)
$v_S(t)$	**1**
$v_{R_C}(t)$	
$v_{R_L}(t)$	

4. Using an Oscilloscope, make measurements across the two separate resistors and complete **Table 3.3**.
   a. Leaving Channel 1 connected, connect Channel 2 of the oscilloscope across the voltage source (function generator) as shown in **Fig. 3.3B**. Compare the two signals on the oscilloscope relative to the time scale and measure R_L's time shift (***ts***). Convert the ***ts*** with Equation 3.6.

$$\varphi = 2\pi f t_s \tag{3.6}$$

   b. Leaving Channel 2 connected, move Channel 1 of the oscilloscope across the resistor in series with the capacitor as shown in **Fig. 3.3A**. Compare the two signals on the oscilloscope relative to the time scale and measure R_C's time shift ***ts***.

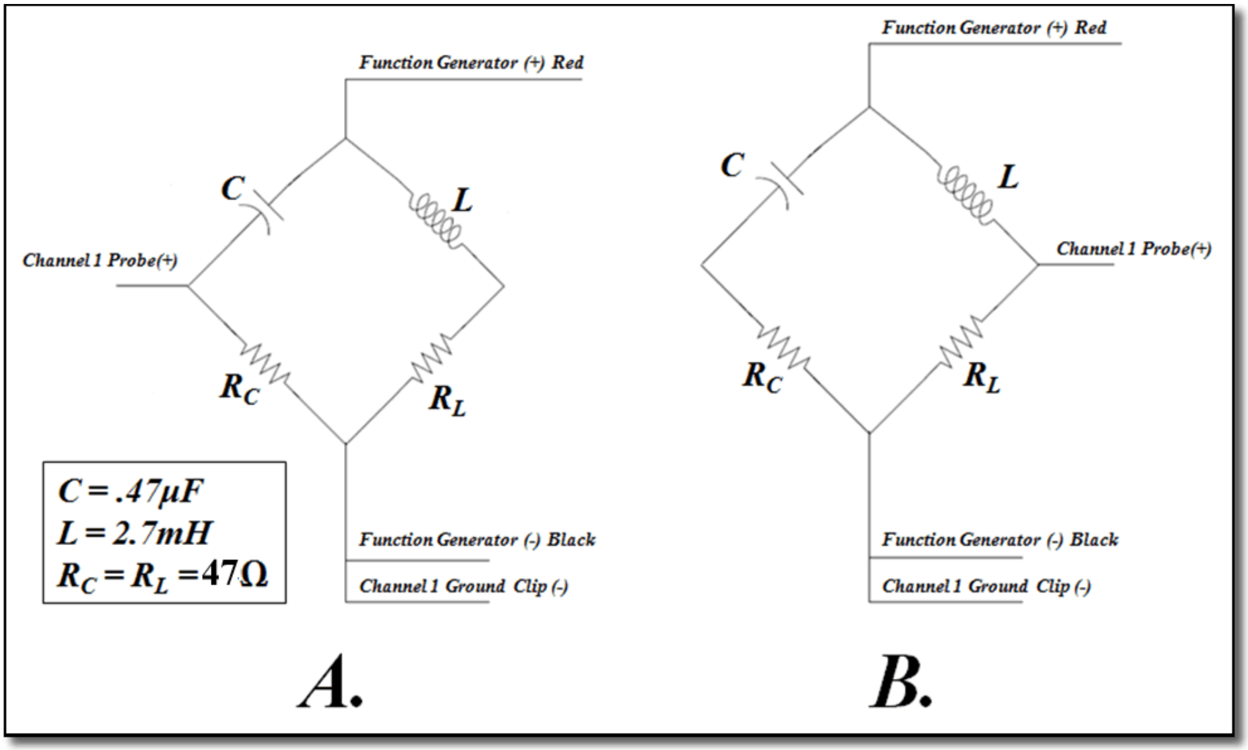

Figure 3.3: Signals Compared to the Function Generator.

Table 3.3 Sinusoid Phase Angle Measurements

Signal	t_s(*sec*)	φ (*rad*)	φ (*degrees*)
$v_S(t)$	0	0	0
$v_{R_L}(t)$			
$v_{R_C}(t)$			

5. Attach hand calculations for **Tables 3.1** and **3.3** at the end of the lab.

6. Using values from the calculations made in **Tables 3.1** and **3.3**, plot the sinusoidal equations for $v_{R_L}(t)$, $v_{R_C}(t)$, and $v_S(t)$ on the same graph using MATLAB.

3.4 Lab Requirements

1. Complete Tables 3.1, 3.2, and 3.3. (2 points each, 6 total)
2. Write an abstract for this lab and submit. (2points)
3. Show hand calculations for Tables 3.1 and 3.3. Insert after this page. (2 points each, 4 total)
4. Draw the sinusoids by hand from Table 3.1. Label amplitude and period. Insert after this page. (2 points)
5. Generate MATLAB plots of $v_{R_L}(t)$, $v_{R_C}(t)$, and $v_S(t)$ (all three on the same graph) for the calculated values and insert after this page.(2 points)
6. Write out the equations of the sinusoids using Tables 3.2 and 3.3. (2 points each, 4 total)
   a. $v_{R_C}(t) = V_{RC}\cos(2\pi f t + \varphi_{RC})$
   b. $v_{R_L}(t) = V_{RL}\cos(2\pi f t + \varphi_{RL})$
7. Using Kirchhoff's Voltage Law it is possible to find $v_C(t)$ and $v_L(t)$ by subtracting the individual voltage drops from the voltage source. Solve each problem below. (4 points each, 8 total)
   a. $v_S(t) - v_{R_C}(t) = V_C\cos(2\pi f t + \varphi_C)$
   b. $v_S(t) - v_{R_L}(t) = V_L\cos(2\pi f t + \varphi_L)$

Page intentionally blank

Page intentionally blank

Laboratory 4

Two Loop Circuit Application of Systems of Equations

4.1 Laboratory Objective

The objective of this laboratory is to learn the basics of systems of equations and matrices and their application in engineering.

4.2 Educational Objective

After performing this experiment, students should be able to:

1. Solve for the unknowns by use of a matrix inverse, Cramer's Rule, substitution, and MATLAB.

4.3 Background

Simultaneous equation solving is a key skill in many engineering applications. For example, software in finite element modeling and thermodynamics solve multiple equations with multiple variables. MATLAB is one such piece of software that can solve simultaneous equations. The following methods will solve relatively small problems easily and can provide an answer quickly, however with larger systems, hand calculations can be exhaustive and software becomes the more intelligent route to the solution.

4.3.1 Problem Statement

A system of equations can be written as $A\vec{x} = \vec{b}$, where A is the coefficient matrix, $\overrightarrow{x}$ is a vector of unknowns, and $\overrightarrow{b}$ is a vector of the right hand sides of the equations. For illustration, the following matrices will be used for explaining the methods.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\overrightarrow{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\overrightarrow{b} = \begin{bmatrix} m \\ n \end{bmatrix}$$

4.3.2 Matrix Inverse Method

$$\overrightarrow{x} = A^{-1}\overrightarrow{b}$$

$$A^{-1} = \frac{1}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \tag{4.1}$$

4.3.3 Cramer's Rule

$$x = \frac{\begin{vmatrix} m & b \\ n & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a & m \\ c & n \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

4.3.4 Substitution

The system $A\overrightarrow{x} = \overrightarrow{b}$ can be written as two equations.

$$ax + by = m \tag{4.2}$$

$$cx + dy = n \tag{4.3}$$

Solve for x in Equation 4.3 and substitute into Equation 4.2.

$$a(\frac{n - dy}{c}) + by = m$$

$$y = \frac{m - \frac{an}{c}}{b - \frac{ad}{c}}$$

Once y is known, substitute back into either Equation 4.2 or 4.3, and solve for x.

4.3.5 MATLAB

MATLAB will solve the problem $A\overrightarrow{x} = \overrightarrow{b}$ by two methods.

$$\overrightarrow{x} = A^{-1}\overrightarrow{b}$$

$$\overrightarrow{x} = A \backslash \overrightarrow{b}$$

NOTE: The second method is pronounced “A left division b” and is much more efficient for larger matrices than method one.

4.4 Procedure

1. The currents flowing through loop 1 and loop 2 in the circuit below are unknown. Construct the circuit and use the lab equipment to find these currents. Complete Table 4.1.

   NOTE: $R_1 = R_2 = 100\Omega$ and $R_3 = R_4 = 200\Omega$.

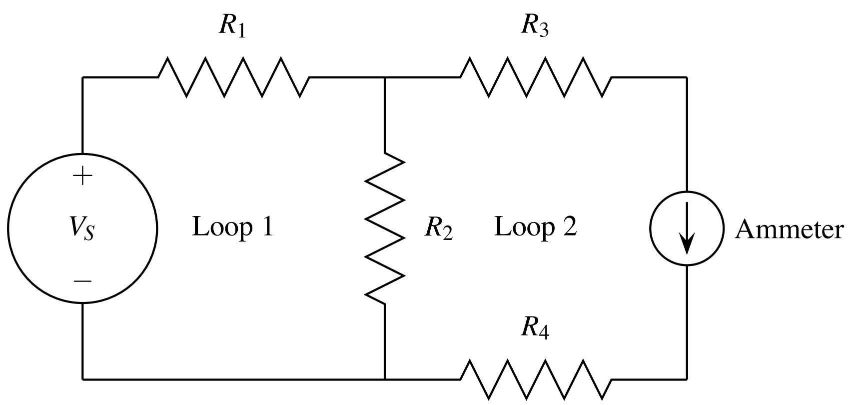

Figure 4.1: A Two-Loop Circuit

Table 4.1 Two-Loop Circuit

V_S (Volts)	I_1 (Amps)	I_2 (Amps)
5		
7		

4.5 Lab Requirements

1. Complete Table 4.1. (2 points)
2. Write an abstract for this lab. Insert after this page. (Writing grade: 2 points)
3. By hand, calculate the unknown currents, I_1 and I_2 using the three methods below and attach after this sheet. (2 points each)

   a) Inverse Matrix Method

   b) Cramer's Rule

   c) Substitution

   NOTE: Use the following matrix setup (Take V_S to be 7):

$$\begin{bmatrix} (R_1+R_2) & -R_2 \\ -R_2 & (R_2+R_3+R_4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \end{bmatrix}$$

4. Write a MATLAB code implementing the Inverse Matrix Method. Have the user input values for all resistors and the voltage source. Check your answer with "Left Division". Attach after this sheet: (4 points) (both cases where $V_S = 5$and 7 V)

   a) m-file

   b) output

5. Compare your calculated values for I_1 and I_2 with the measured values. Why are they different? (2 points)

Laboratory 5

Freefall Application of the Derivative

5.1 Laboratory Objective

The objective of this laboratory is to illustrate the application of a derivative with a freefall exercise.

5.2 Educational Objective

After performing this experiment, students should be able to:

1. Understand the relationship between position, velocity, and acceleration.
2. Identify the key parameters of freefall.
3. Use MATLAB symbolics to calculate derivatives.

5.3 Background

The derivative is a tool that describes the rate of change of a quantity with respect to the change in another. Geometrically this is equivalent to slope.

5.3.1 Position, Velocity, and Acceleration

Given a function $y(t)$ that represents position with respect to time, one can derive the expressions for the velocity $v(t)$ and the acceleration $a(t)$. Velocity is simply the derivative of $y(t)$ with respect to time and acceleration is the second derivative of $y(t)$ with respect to time.

$$v(t) = \frac{dy}{dt}$$

$$a(t) = \frac{d^2y}{dt^2} = \frac{dv}{dt}$$

Velocity can also be calculated using $\frac{\Delta y}{\Delta t}$, or

$$v(t) = \frac{y_2 - y_1}{t_2 - t_1}$$

Similarly, acceleration can be calculated using $\frac{\Delta v}{\Delta t}$, or

$$a(t) = \frac{v_2 - v_1}{t_2 - t_1}$$

The freefall apparatus used in this lab consists of a free fall device, an ultrasound sensor which is mounted to fixed position, and a computer to record the data.

5.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the laboratory equipment.

1. Open Data Studio and click the Setup icon.
2. Set the sample rate to 20 Hz. (ie. $\Delta t = 1/20\text{sec}$)
3. Close the setup window.
4. Start data collection and drop device.
5. Press stop after the object hits the ground.
6. Copy data into Microsoft Excel by dragging a box around the data and then copy/pasting.
7. Construct Table 5.1 in Excel and plot the measured position, velocity, and acceleration vs. time.

Table 5.1 Position, Velocity, and Acceleration

$y_i\ (m)$	$t_i\ (s)$	$\Delta y = y_{i+1} - y_i\ (m)$	$\Delta t = t_{i+1} - t_i\ (s)$	$v_i = \frac{\Delta y}{\Delta t}\ (\frac{m}{s})$	$\Delta v = v_{i+1} - v_i(\frac{m}{s})$	$a_i = \frac{\Delta v}{\Delta t}\ (\frac{m}{s^2})$
0	0	-	-	0	-	0
y_1	t_1	$y_1 - y_0$	$t_1 - t_0$	v_1	$v_1 - v_0$	a_1
y_2	t_2	$y_2 - y_1$	$t_2 - t_1$	v_2	$v_2 - v_1$	a_2
etc.	etc.	etc.	etc.	etc.	etc.	etc.

8. Repeat this procedure for 40 Hz and 50 Hz sparks.

5.5 Lab Requirements

1. Write an abstract for this lab. Insert after this page. (Writing grade: 2 points)
2. Insert all three Tables after this page. (2 points each)
   a) 20 Hz
   b) 40 Hz
   c) 50 Hz
3. Insert the measured position, velocity, and acceleration plots for the best (where acceleration is closest to 9.81 m/s^2 case after this page. (Don't forget to properly label the graphs!) (2 points each)
4. Write a MATLAB script that will plot the theoretical (calculated) position, velocity, and acceleration vs. time. Use MATLAB symbolics and subplot commands. (2 points each)

   NOTE: For freefall, $y(t) = y_0 + v_0 t + \frac{1}{2}at^2\,(m)$.
5. Answer the following questions:
   a) In freefall what physical quantity does the acceleration represent? (2 points)

   b) What is the mathematical relationship between position, velocity, and acceleration? (2 points)

Laboratory 6

Spring Work Application of the Integral

6.1 Laboratory Objective

The objective of this laboratory is to illustrate the application of an integral with an exercise with spring work.

6.2 Educational Objective

After performing this experiment, students should be able to:

1. Understand that geometrically, an integral calculates area under a curve.
2. Understand the work done on a spring.

6.3 Background

Work is a fundamental concept of many physical systems. In general, the sum of all forces over a given distance is work.

$$W = \int_0^x F(x)dx \tag{6.1}$$

A spring has work done on it when it is stretched. The spring force is linearly related to the distance stretched by a constant k.

$$F(x) = kx \tag{6.2}$$

The work done on a spring by a mass can be found by substituting Equation 6.2 into Equation 6.1.

$$W = \int_0^x kx\,dx \tag{6.3}$$

Figure 6.1 below shows the setup that will be used in the lab.

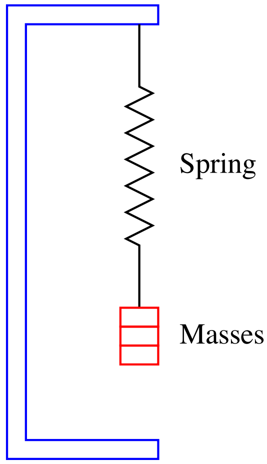

Figure 6.1: Spring & Mass System

6.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the laboratory equipment.

1. Attach the spring to the stand and suspend the mass hanger from the other end. Record the measurement from the scale. This is your reference value x_o.
2. Complete Tables 6.1, 6.2, and 6.3.
   a) Add mass as shown in each table and record the measurement from the scale.
   b) Calculate the force on the spring in Newtons. ($g = 9.81\, m/s^2$)

   $$F = mg$$

   c) Calculate Δx due to each added mass.

   NOTE: Remember to take the absolute value of your answers in column four if they are negative.

Table 6.1

Mass (kg)	Scale (m)	F (N)	$\Delta x = \lvert x_i - x_0 \rvert$ (m)	$base = \lvert x_i - x_{i-1} \rvert$
0	$x_0 =$	0	-	-
0.08	$x_1 =$			
0.16	$x_2 =$			

Table 6.2

Mass (kg)	Scale (m)	Force (N)	$\Delta x = \|x_i - x_0\|$ (m)	$base = \|x_i - x_{i-1}\|$
0	$x_0 =$	0	-	-
0.04	$x_1 =$			
0.08	$x_2 =$			
0.12	$x_3 =$			
0.16	$x_4 =$			

Table 6.3

Mass (kg)	Scale (m)	Force (N)	$\Delta x = \|x_i - x_0\|$ (m)	$base = \|x_i - x_{i-1}\|$
0	$x_0 =$	0	-	-
0.02	$x_1 =$			
0.04	$x_2 =$			
0.06	$x_3 =$			
0.08	$x_4 =$			
0.1	$x_5 =$			
0.12	$x_6 =$			
0.14	$x_7 =$			
0.16	$x_8 =$			

3. Plot Force vs. Δx in MATLAB for all three Tables.
4. Using MATLAB's Basic Fitting tool, find the slope and y-intercept of the graphs.
5. Plot a bar graph of Force vs. Δx in MATLAB for all three Tables.

   NOTE: MATLAB syntax is given in Section 6.6.

6.5 Lab Requirements

1. Complete Tables 6.1, 6.2, and 6.3. (2 points each)
2. Write an abstract for this lab. Insert after this page. (Writing grade: 2 points)
3. Insert 3 linear plots after this page. (2 points each)
4. Insert 3 bar graphs after this page. (2 points each)
5. Write a MATLAB script that calculates the area of the bar graphs. Insert after this page. (2 points each)
   a) m-file
   b) output
6. Write a MATLAB script that will calculate the integral of Equation 6.3. Use MATLAB symbolics. (2 points each)

   NOTE: Use the value of k that was found from Table 6.3 in the equation. The limits of integration are 0 to $\Delta x = |x_8 - x_0|$ from Table 6.3.

   a) m-file
   b) output
7. Calculate this integral by hand. (2 points)

   NOTE: Use the value of k that was found from Table 6.3 in the equation. The limits of integration are 0 to $\Delta x = |x_8 - x_0|$ from Table 6.3.

$$W = \int_0^{\Delta x} kx\,dx$$

8. Answer the following questions:
   a) What does the slope of the linear plots physically represent? (2 points)

   b) What are the units of the spring constant k? (2 points)

c) What do the areas of the bar graphs physically represent? (2 points)

d) How do the areas of the bar graphs compare to your answer for Question 6?

6.6 MATLAB Commands

bar(x,y,1) Draws a bar graph with x values at the midpoint of each rectangle.

Laboratory 7

Leaking Bucket Application of a First Order Differential Equation

7.1 Laboratory Objective

The objective of this laboratory is to learn about first order differential equation and its application to a leaking bucket.

7.2 Educational Objectives

After performing this experiment, students should be able to:

1. Understand the modeling of a leaking bucket dynamic system.
2. Measure the key parameters of a leaking bucket dynamic system.
3. Validate a mathematical model of the leaking bucket with observed data.

7.3 Background

Differential equations are an integral part of engineering. Almost all system response can be described by a differential equation. Knowledge of how to solve these problems is key to an engineer's success. This lab looks at one classification of a differential equation; first order, constant coefficient, and homogeneous.

7.3.1 The Leaking Bucket

The system shown in Figure 7.1 can be described by investigating the behavior of the water.

The following equation describes the volumetric flow rate, Q of the system.

$$Q_{in} - Q_{out} - Q_{stored} = 0$$

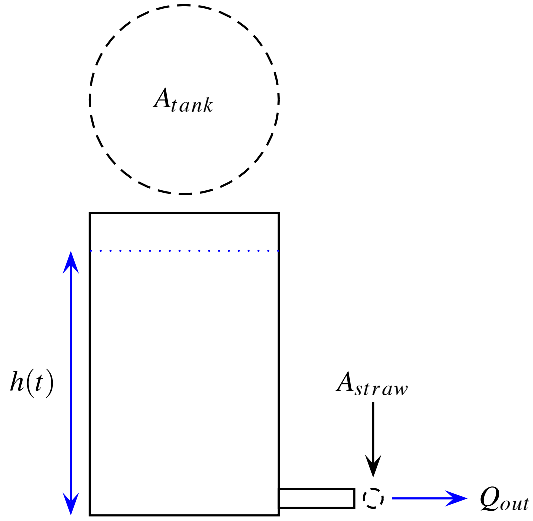

Figure 7.1: Leaking Bucket

There will not be any water flowing into our system, therefore $Q_{in} = 0$.

$$Q_{stored} = -Q_{out}$$

The volumetric flow rate is found by multiplying the velocity by the area.

$$A_{tank}\dot{h} = -A_{straw}v$$

From fluids, the velocity of the water coming out of the straw is $\sqrt{2gh}$.

$$A_{tank}\dot{h} = -A_{straw}\sqrt{2gh}$$

Rearranging terms and writing all constants as one:

$$\begin{aligned} A_{tank}\dot{h} + A_{straw}\sqrt{2g}\sqrt{h} &= 0 \\ A_{tank}\dot{h} + K\sqrt{h} &= 0 \end{aligned}$$

The above equation cannot be solved using the methods of this class because the h on the second term of the equation is under a square root. To accommodate this, the governing equation that will be solved in this lab will be approximated without the square root:

$$A_{tank}\dot{h} + Kh = 0$$

The solution to the governing equation is:

$$h(t) = Ce^{-(K/A_{tank})t} \tag{7.1}$$

Where C is the initial height of the water and the system time constant is defined as $\tau = A_{tank}/K$.

7.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the laboratory equipment.

1. Using the hot glue gun in the lab, glue the straw into the two liter pop bottle near the base.

   NOTE: The straw must be horizontal.

2. Place a piece of tape axially along the bottle from the straw to where the bottle starts to curve near the top.
3. Fill the bottle with water up to where the bottle starts to curve while blocking the straw so that water does not leak out.
4. Place a mark on the tape to indicate the initial height of the water.
5. Release the straw and allow the water to flow out into a drain pan.
6. On the tape, mark the height of the water level every five seconds until water drips from the straw.
7. Remove tape and complete Table 7.1.

Table 7.1

Full Straw			Half Straw		
Time t (sec)	Height h (m)	$\ln(h)$	Time t (sec)	Height h (m)	$\ln(h)$
0	h_1	$\ln(h_1)$	0	h_1	$\ln(h_1)$
5	h_2	$\ln(h_2)$	5	h_2	$\ln(h_2)$
10	h_3	$\ln(h_3)$	10	h_3	$\ln(h_3)$
etc.	etc.	etc.	etc.	etc.	etc.

8. Using Microsoft Excel, plot h vs. t for both straws.
9. Using Microsoft Excel, plot $\ln(h)$ vs. t for both straws.

   a) Fit a line to the data and place the equation on the plot.

   NOTE: The slope of this straight line is $-K/A_{tank}$. The time constant τ is simply the negative inverse of the slope.

7.5 Lab Requirements

1. Write an abstract for this lab. Insert after this page. (Writing grade: 2 points)
2. Complete Table 7.1 and insert after this page. (2 points)
3. Insert two plots of h vs. t after this page. (2 points each)
4. Insert two plots of $\ln(h)$ vs. t after this page. (2 points each)
5. Derive by hand the equation of the straight line for the "ln plot" in terms of C, K, and A_{tank}. (2 points)

   HINT: Start by taking the natural log of both sides of Equation 7.1 and algebraically simplify

6. Answer the following questions:

   a) What is the time constant for the full straw? (Don't forget units.) (2 points)

   b) What is the time constant for the half straw? (Don't forget units.) (2 points)

   c) What type of energy is stored in the water? (2 points)

Laboratory 8

Spring-Mass Application of a Second Order Differential Equation

8.1 Laboratory Objective

The objective of this laboratory is to model spring-mass behavior with a second order differential equation.

8.2 Educational Objective

After performing this experiment,students should be able to:

1. Apply principles of modeling and analysis to a spring-mass system.
2. Identify and measure the key parameters of a spring-mass system.
3. Validate a mathematical model (differential equation) with measured data.

8.3 Background

Another class of differential equations are second order applications. These equations contain a second derivative of variable in question. In the case of a spring-mass system, the displacement as a function of time is the unknown quantity.

8.3.1 The Spring-Mass System

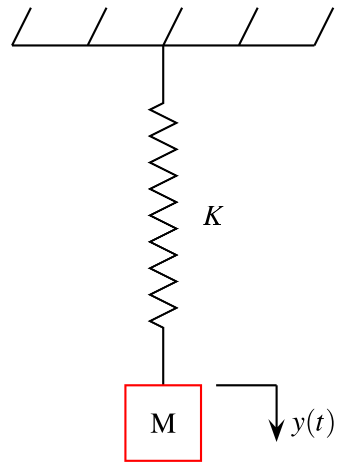

Figure 8.1: Spring-Mass System

The spring-mass system shown in Figure 8.1 has kinetic energy associated with the mass moving up and down and potential energy stored in the spring. This energy is passed back and forth as the spring oscillates.

The free body diagram (FBD) in Figure 8.2 shows all forces acting on the mass.

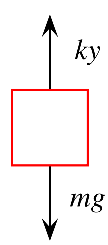

Figure 8.2: Free Body Diagram of Spring-Mass System

From equilibrium of forces in the y-direction,

$$k\delta = mg,$$

which gives

$$\delta = \frac{mg}{k}.$$

This represents the static deflection of the spring. Once the mass is displaced from the equilibrium position and allowed to vibrate, the mass-spring system is no longer in equilibrium. Applying Newtons Second Law and simplifying:

$$\begin{aligned}
\Sigma F &= ma \\
mg - k[\delta + y(t)] &= m\ddot{y}(t) \\
mg - k\delta - ky(t) &= m\ddot{y}(t) \\
mg - k\left(\frac{mg}{k}\right) - ky(t) &= m\ddot{y}(t) \qquad (8.1) \\
m\ddot{y}(t) + ky(t) &= 0 \qquad (8.2)
\end{aligned}$$

Equation 8.1 is the governing equation of a frictionless spring-mass system.

The solution to this equation is:

$$y_{total}(t) = A\cos\left(\sqrt{\frac{k}{m}}t\right)$$

The mass will oscillate as a cosine wave with amplitude A and angular frequency $\sqrt{\frac{k}{m}}$.

8.4 Procedure

Follow the steps outlined below after the Lab Teaching Assistant has explained how to use the laboratory equipment.

1. Attach the spring to the stand and suspend the mass hanger from the other end. Record the measurement from the scale. This is your reference value.
2. Complete Table 8.1 using one spring and two springs in series.

   NOTE: Use two of the same springs for the Double Spring measurements.

Table 8.1

Single Spring		Double Spring	
Mass (kg)	y_i (m)	Mass (kg)	y_i (m)
0		0	
0.25		0.25	
$k_1 =$		$k_2 =$	

3. Calculate the spring constants k_1 and k_2 with the following equation.

$$k_i = \left|\frac{g(m_1 - m_2)}{y_1 - y_2}\right|$$

4. Place a mass on the hanger, displace it, and measure the time, t it takes to complete 20 cycles according to Table 8.2.

5. Calculate the period of oscillation $T_{measured}$ using Equation 8.3.

$$T_{measured} = \frac{t_{20}}{20} \tag{8.3}$$

Table 8.2

One Spring		Two Springs		One Spring		Two Springs	
$m = 0.15$ kg				$m = 0.25$ kg			
t_{20} (sec)	$T_{measured}$ (sec)	t_{20} (sec)	$T_{measured}$ (sec)	t_{20} (sec)	$T_{measured}$ (sec)	t_{20} (sec)	$T_{measured}$ (sec)

6. The theoretical period T_{calc} can be calculated by Equation 8.4. Find the calculated period for all cases by completing Table 8.3.

$$T_{calc} = 2\pi\sqrt{\frac{m}{k}} \tag{8.4}$$

Table 8.3

k_1	k_2	k_1	k_2
$m = 0.15$ kg		$m = 0.25$ kg	
T_{calc}	T_{calc}	T_{calc}	T_{calc}

8.5 Lab Requirements

1. Complete Tables 8.1, 8.2, and 8.3. (2 points each)
2. Write an abstract for this lab. Insert after this page. (Writing grade: 2 points)
3. Show calculations for all tables and insert after this page. (2 points each)
4. Answer the following question:
    a) Compare $T_{measured}$ with T_{calc}. Why are they different? (2 points)